PROBABILITY OF FORMATION OF COATINGS OF THERMALLY ACTIVE PARTICLES

V. P. Bushlanov

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The strength of a coating can be increased if deposition is carried out in such a way that the constituent particles are deposited on top of previously deposited particles that are still cooling (are thermally active) [1]. The probability conditions for the formation of coatings of thermally active coatings were calculated in [2] with the assumption that N previously deposited particles are not in contact with one another.

In the present study, we calculate the probability P_0 throughout a range of rates of flow G (kg/sec). It is shown that P_0 is expressed linearly through the porosity of the coating and depends on the dimensionless parameter $E_0^2 = NR^2/\rho^2$ (where R is the radius of the disk of an adherent particle and ρ is the radius of the deposition spot).

1. In plasma-spray deposition, an attempt is made to use particles of similar dimensions. The radii R_0 of spherical particles equal in volume to the deposited particles lie within the range 10-100 μm, while $\rho \sim 10^{-2}$ m. Thus $R_0^2/\rho^2 \sim 10^{-6}$-$10^{-4}$. As in [2], we will assume that the coating is formed of disks of radius R and height h such that

$$4\pi R_0^3/3 = \pi R^2 h,\ E^2 = R^2/\rho^2 \ll 1. \tag{1.1}$$

Let p(x, y) be the probability density of the location of the center of mass of a particle being deposited at the point of the substrate with the coordinates (x, y). The total probability of interaction with previous N particles which remain thermally active has the form [2]

$$P(N) = \int\int_{-\infty}^{\infty} d\xi_1\, d\eta_1 \dots \int\int_{-\infty}^{\infty} d\xi_N\, d\eta_N \prod_{i=1}^{N} p(\xi_i, \eta_i) \int\int_{Q(\xi_1,\eta_1,\dots,\xi_N,\eta_N)} p(x, y)\, dx\, dy. \tag{1.2}$$

Here $\xi_1, \eta_1, \dots, \xi_N, \eta_N$ are the coordinates of the centers N of disks of radii R located on the substrate in a thermally active state; Q is the region of the substrate occupied by N disks.

Below, we present an effective method of specifying the region Q and calculating (1.2). We introduce piecewise-constant functions E_i and Σ_i: $(x, y) \in C_R(\xi_i, \eta_i)$, $i = 1, 2, \dots, N$, where $C_R(\xi_i, \eta_i)$ is a circle of radius R with the center (ξ_i, η_i), while $E_i = 0$ outside this circle, $\Sigma(x, y) = 1 - \prod_{k=1}^{N}(1 - E_k)$. It is evident that $\Sigma = 1$ inside Q and $\Sigma = 0$ outside Q. We rewrite (1.2) by means of Σ:

$$P(N) = \int\int_{-\infty}^{\infty} d\xi_1\, d\eta_1 \dots \int\int_{-\infty}^{\infty} d\xi_N\, d\eta_N \prod_{i=1}^{N} p(\xi_i, \eta_i) \int\int_{-\infty}^{\infty} \Sigma p(x, y)\, dx\, dy. \tag{1.3}$$

Changing the order of integration in (1.3), we obtain

$$P(N) = 1 - \int\int_{-\infty}^{\infty} p(x, y)\, [1 - I(x, y)]^N\, dx\, dy; \tag{1.4}$$

$$I(x, y) = \int\int_{-\infty}^{\infty} E_i p(\xi_i, \eta_i)\, d\xi_i\, d\eta_i = \int\int_{C_R(x,y)} p(\xi_i, \eta_i)\, d\xi_i\, d\eta_i. \tag{1.5}$$

2. Since $p(x, y) < 1$, we conclude from (1.5) that $I(x, y) < \pi\rho^2 E^2$. With a high degree of accuracy, we find from (1.1), (1.4) that

$$P(N) = \int\int_{-\infty}^{\infty} p(x, y) \exp(-IE_0^2/E^2)\, dx\, dy,\ E_0^2 = E^2 N. \tag{2.1}$$

Let us calculate P(N) for the case when the probability of the incidence of a particle in the deposition spot is the same for all (x, y). From (1.4)-(1.5), (2.1) we have

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$$I = E^2,\ P(N) = 1 - (1 - E^2)^N \cong 1 - \exp(-E_0^2) = P_0(E_0^2). \tag{2.2}$$

Now we take $p(x, y) = p_0(x, y) = \exp-[(x^2 + y^2)/\rho^2]/(\pi\rho^2)$ from [2]. From (1.4) we obtain

$$P(N) = \sum_{j=1}^{N} P_j,\ P_j = (-1)^{j+1} J_j N!/(j!\,(N-j)!),\ J_j = \int\int_{-\infty}^{\infty} p(x, y)\, I^j(x, y)\, dx\, dy. \tag{2.3}$$

We calculate the exact value of P_1. It should be noted that P_1 agrees with the analogous quantity obtained by approximation in [2]. We substitute the variables of integration as follows in (1.5)

$$\begin{gathered} \xi = \xi_i - x,\ \eta = \eta_i - y;\ x = r_1 \cos\theta,\ y = r_1 \sin\theta; \\ \xi = r\cos\theta,\ \eta = r\sin\theta;\ \varepsilon_1 = r_1/\rho,\ \varepsilon = r/\rho. \end{gathered} \tag{2.4}$$

We find from (1.5) and (2.4) that

$$\begin{gathered} I(x, y) = I_1(E, \varepsilon_1) = \exp(-\varepsilon_1^2) \int_0^E I_0(-2\varepsilon\varepsilon_1) \exp(-\varepsilon^2)\, d\varepsilon^2, \\ I_0(-2\varepsilon\varepsilon_1) = \int_0^{2\pi} \exp(-2\varepsilon\varepsilon_1 \cos\varphi)\, d\varphi. \end{gathered} \tag{2.5}$$

It is evident from (2.5) that I_0 is a modified zeroth-order Bessel function. Thus,

$$I_0(-2\varepsilon\varepsilon_1) = \sum_{k=0}^{\infty} (\varepsilon\varepsilon_1)^{2k}/(k!)^2. \tag{2.6}$$

Using a formula for expansion $\exp(E^2)$ into a series, we find from (2.3), (2.5), and (2.6) that

$$P_1/N = 1 - \exp(-E^2/2). \tag{2.7}$$

To within terms of the order of $O(E^4)$, we obtain the following from (2.7)

$$P_1 = E_0^2/2. \tag{2.8}$$

Particles which land outside the deposition spot rebound from the substrate. It is therefore best if we replace $p_0(x, y)$ by $p(x, y)$, which is nontrivial only within the spot, i.e., $p(x, y) = E^2 p_0(x, y)/\langle p_0(x, y)\rangle = p_0(x, y)/(\pi\rho^2(1 - 1/e))$ if $(x, y) \in C_\rho(0, 0)$. Here, instead of G we need to take βG (where β is the powder use coefficient). In the present case, $\varepsilon_1 \leqslant 1$, and to within terms of the order of $O(E_0^2 E^2)$ we find from (2.1) and (2.5) that

$$P(N) = 1 - (1 - \exp(-E_0^2)) \exp(-E_0^2/(e - 1))/E_0^2. \tag{2.9}$$

3. Let us determine the range of E_0^2 under normal deposition conditions [1]. Suppose that t_a is the time of thermal activity of a deposited particle. Then

$$\beta G t_a = 4\pi R_0^3 \gamma N/3, \tag{3.1}$$

where γ is density; N is the number of particles which adhere to the substrate during the time t_a. In accordance with (2.1), $E_0^2 \sim 10^{-2}$. When $P(N) \sim 1$, a layer of deposited material of the thickness h should be located on the substrate. Thus, $\beta G t_a \sim \pi\rho^2 h\gamma$ and $E_0^2 \sim 1$. It follows from this that during deposition the parameter E_0^2 goes from a value which is much less than unity [and corresponds to very small values of P(N)] to values of the order of unity [which corresponds to $P(N) \sim 1$].

It is interesting to calculate E_0^2 in terms of the values of h and t_a reported in [1] – specifically, the values obtained in [1] using the assumption

$$h = 2R_0 - t_a(1 - \mu)v,\ t_a = h^2/(4\alpha^2 a_1). \tag{3.2}$$

Here, v is the velocity of a drop at the moment of impact; μ is a coefficient which characterizes the stiffness of a particle and depends on the collision velocity and material of the particles; a_1 is the diffusivity of the particle material; $\alpha_1 \leqslant 1$ is a dimensionless parameter. It follows from (2.1), (3.1), and (3.2) that

$$E_0^2 = \frac{\beta G}{\pi\gamma\rho^2}\left[-\frac{1}{2(1-\mu)v} + \sqrt{\frac{1}{(1-\mu)^2 v^2} + \frac{R_0}{2\alpha^2 a_1 (1-\mu) v}}\right]. \tag{3.3}$$

4. We calculated the probability $P_0(E_0^2)$ of the interaction of particles being deposited during the period of their thermal activation. This probability was calculated in the form (2.9) for the real-valued function p(x, y) to within terms of the order of $O(E_0^2 E^2)$; P_0

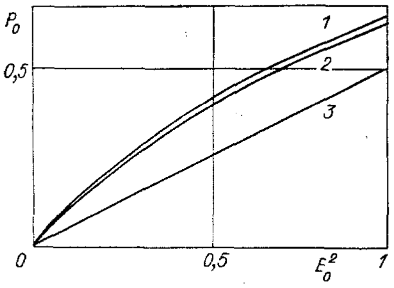

Fig. 1

depends on the parameter E_0^2. We obtained Eq. (3.3) on the basis of Eqs. (3.2) (which were in turn taken from [1]) as an example of deposition parameters which might influence the value of E_0^2.

Figure 1 shows the relations $P_0(E_0^2)$: line 1 corresponds to (2.9), line 2 corresponds to (2.2), and line 3 corresponds to (2.8). Curves 1 and 2 are close despite the substantial difference in the probability densities. It can be suggested on the basis of these two curves and Eq. (2.1) that the relations $P_0(E_0^2)$ will not differ very much from one another for different values of the real probability densities – as occurs in the figure. We should also note that even if P_1 had been calculated exactly in [2] (as we did here), there would still be a significant difference from the actual values of P_0 (see Fig. 1). This, of course, has to do with the fact that the assumption made in [2] that N thermally active particles are not in contact with one another is invalid for $P_0 \sim 1$.

Let us show how (2.9) can be used to optimize the conditions for the process when a coating is deposited on thermally active particles. Proceeding on the basis of the definition of P(N), we have

$$P(N) = 1 - \varepsilon_N. \tag{4.1}$$

Here, ε_N is the relative area of the deposition spot not occupied by N disks – in other words, surface porosity on the substrate in the case of the deposition of N disks on it. For deposition to take place on thermally active particles, it is necessary that N > S, where S is found from Eq. (4.1) with N = S and ε_S is the actual surface porosity of the coating on the substrate. Since P(N) is a monotonically increasing function of the argument N, then for N > S it is necessary that

$$P(N) = 1 - \varepsilon_N = P_0(E_0^2) > P(S) = 1 - \varepsilon_S. \tag{4.2}$$

Inserting (2.9) into (4.2), we obtain the condition of optimality of the coating deposition process:

$$1 - (1 - \exp(-E_0^2)) \exp(-E_0^2/(e-1))/E_0^2 > 1 - \varepsilon_S. \tag{4.3}$$

Instead of (4.3), we can use graph 1 (see Fig. 1). For this, we assign ε_S and calculate the probability $P_0(E_0^2) = 1 - \varepsilon_S$. We then find E_{0*}^2 for this probability on the x axis. Satisfaction of the optimality condition requires that $E_0^2 > E_{0*}^2$. For example, if E_0^2 is calculated from (3.3), then the optimality of the process can be regulated with the parameters β, G, γ, ρ, μ, v, R_0, α, a_1.

It should also be noted that the left side of inequality (4.2) is the theoretical dependence of porosity ε_N on the dimensionless argument E_0^2.

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